Quiz 1

April 15, 2005

Division: **ID#:** **Name:**

1. In $\boldsymbol{Z}_{24}$ find the inverses of [7] and [13].

2. Show that if n is an odd integer, then $n^2 \equiv 1 \pmod 8$.

3. Find the general solution of the congruence $6x \equiv 11 \pmod 5$.

Message: What do you expect from this course? Any requests?

Quiz 2

April 22, 2005

Division: **ID#:** **Name:**

Let $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 8 & 3 & 2 & 7 & 6 & 4 \end{pmatrix}$, $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 6 & 5 & 2 & 1 & 7 & 3 & 4 \end{pmatrix}$.

1. Compute π^{-1}.

2. Compute $\pi\sigma$.

3. Compute $\pi\sigma\pi^{-1}$.

4. Express each of π and σ as a product of disjoint cycles.

5. Determine $\mathrm{sign}(\pi)$ and $\mathrm{sign}(\sigma)$.

Message: Any requests?

Quiz 3

May 6, 2005

Division: **ID#:** **Name:**

1. Let S be the subset of $\boldsymbol{R}\times\boldsymbol{R}$ specified below and define $(x,y)*(x',y')=(x+x',y+y')$. Say in each case whether $(S,*)$ is a semigroup, a monoid, a group, or none of these.

   (a) $S=\{(x,y)\mid x+y\geq 0\}$;

   (b) $S=\{(x,y)\mid x+y>0\}$;

   (c) $S=\{(x,y)\mid |x+y|\geq 1\}$;

   (d) $S=\{(x,y)\mid 2x+3y=0\}$.

2. Let $(M,\circ)$ be a monoid with an identity element e, i.e., for every $x\in M$, $x\circ e=x=e\circ x$.

   (a) Show that if $a,b,c\in M$ satisfy $a\circ b=e=b\circ c$, then $a=c$.

   (b) Suppose $a,b,c\in M$ satisfy $a\circ b=e=b\circ c$ as above. Then $a\circ x=a\circ y$ for $x,y\in M$ implies $x=y$.

   (c) Suppose for every $x\in M$ there is an element $y\in M$ such that $x\circ y=e$. Then $(M,\circ)$ is a group.

Message: Any requests or questions?

Quiz 4

May 13, 2005

Division: **ID#:** **Name:**

In each case say whether or not S is a subgroup of the group G:

1. $G = GL_n(\boldsymbol{R})$, $S = \{A \in G \mid \det(A) = 1\}$.

2. $G = (\boldsymbol{R}, +)$, $S = \{x \in \boldsymbol{R} \mid |x| \leq 1\}$.

3. $G = \boldsymbol{R} \times \boldsymbol{R}$, $S = \{(x, y) \mid 3x - 2y = 1\}$: here the group operation adds the components of ordered pairs.

4. $G = (\boldsymbol{Z}_6, +)$, $S = \{[0], [1], [5]\}$: here the usual addition of congruence classes is used.

5. $G = (\boldsymbol{Z}_{11}^*, \cdot)$, $S = \{[1], [7]\}$: here $\boldsymbol{Z}_{11}^*$ is the set of invertible congruence classes $[a]$ modulo 11, i.e., such that $gcd\{a, 11\} = 1$, and multiplication of congruence classes is used.

Message: Any questions or requests?

Quiz 5

May 25, 2005

Division: **ID#:** **Name:**

1. Let H be a subgroup of a gourp G.

   (a) For $x \in G$, let $\ell_x : G \to G$, $(y \mapsto xy)$. Show that ℓ_x is a bijection.

   (b) For $x, y \in G$, show that

   $$xH = yH \Leftrightarrow x^{-1}y \in H.$$

   (c) For $x, y \in G$, show that

   $$xH = yH \Leftrightarrow Hx^{-1} = Hy^{-1}.$$

2. Let $G = \boldsymbol{Z}_8^*$ be a multiplicative group consisting of invertible congruence classes $[a]$ modulo 8, i.e., $\boldsymbol{Z}_8^* = \{[1], [3], [5], [7]\}$. Let $H = \langle [3] \rangle$ and $K = \langle [5] \rangle$.

   (a) Find a subgroup L in G satisfying the following.

   $$\langle H, K \rangle \cap L \neq \langle H \cap L,\ K \cap L \rangle.$$

   (b) Using your choice of L in the previous problem, check whether the following holds or not.

   $$(HK) \cap L = (H \cap L)K.$$

Message: Any questions or requests?

Quiz 6

June 1, 2005

Division: **ID#:** **Name:**

1. Let H be a subgroup of a group G. Show the following.

$$xhx^{-1} \in H \text{ for all } h \in H,\ x \in G \Rightarrow xH = Hx \text{ for all } x \in G.$$

2. Let $G = GL_2(\boldsymbol{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \ a, b, c, d \in \boldsymbol{R},\ ad - bc \neq 0 \right\}$,

$$B = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \middle| \ a, b, d \in \boldsymbol{R},\ ad \neq 0 \right\} \text{ and } U = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \middle| \ b \in \boldsymbol{R} \right\}.$$

   (a) Show that U is a normal subgroup of B.

   (b) Show that U is not a normal subgroup of G.

Message: Any questions or requests?

Quiz 7

June 8. 2005

Division: **ID#:** **Name:**

Let $\alpha : G \to H$ be a group homomorphism. Prove the following.

1. $\alpha(1_G) = 1_H$.

2. $\alpha(x^{-1}) = \alpha(x)^{-1}$ for all $x \in G$.

3. For all $n \in \boldsymbol{Z}$ and $x \in G$, $\alpha(x^n) = \alpha(x)^n$.

4. If $K \leq G$, then $\alpha(K) \leq H$.

5. If $N \lhd H$, then $\alpha^{-1}(N) \lhd G$.

Message: Any questions or requests?

Quiz 8

June 15, 2005

Division: **ID#:** **Name:**

Let G be a group and $\alpha : G \times G \to G$ $((g, x) \mapsto gxg^{-1})$.

1. Show that α defines a left action of G on itself.

2. For $x \in G$, show that $\mathrm{St}_G(x) = \{g \mid (g \in G) \wedge (\alpha(g, x) = x)\}$ is a subgroup of G.

3. For $g \in G$, let $\mathrm{Fix}(g) = \{x \mid (x \in G) \wedge (\alpha(g, x) = x)\}$. Show that $\mathrm{Fix}(g) = \mathrm{St}_G(g)$, where $\mathrm{St}_G(g)$ is the subgroup defined in the previous problem.

4. Suppose $G = S_4$, the symmetiric group of degree 4. Let $\sigma = (1, 2, 3, 4)$. Find all elements in $\mathrm{St}_G(\sigma)$.

5. Let G and σ be as above. How many elements are there in $\{\alpha(\tau, \sigma) \mid \tau \in G\}$?

Message: Any questions or requests?