Linear Algebra I *November 16, 2007*

Final Exam 2007

(Toal: 100pts)

Division: **ID#:** **Name:**

1. Find the values of α and β for the following system to have a solution. (10 pts)

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 6 \\ 6x_1 + 7x_2 + 8x_3 + 9x_4 + 10x_5 &= 11 \\ 11x_1 + 12x_2 + 13x_3 + 14x_4 + 15x_5 &= \alpha \\ 16x_1 + 17x_2 + 18x_3 + 19x_4 + 20x_5 &= \beta \end{cases}$$

Points:

1.	2.	3.(a)	(b)	(c)	4.(a)	(b)	5.(a)	(b)	(c)	6.(a)	(b)	**Total**

2. Show the following. (10 pts)

$$\begin{vmatrix} 1 & x_1 & {x_1}^2 & \cdots & {x_1}^{n-1} \\ 1 & x_2 & {x_2}^2 & \cdots & {x_2}^{n-1} \\ 1 & x_3 & {x_3}^2 & \cdots & {x_3}^{n-1} \\ & & \cdots\cdots\cdots & & \\ 1 & x_n & {x_n}^2 & \cdots & {x_n}^{n-1} \end{vmatrix} = (x_n - x_1)(x_n - x_2)\cdots(x_n - x_{n-1}) \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} \\ 1 & x_2 & {x_2}^2 & \cdots & {x_2}^{n-2} \\ 1 & x_3 & {x_3}^2 & \cdots & {x_3}^{n-2} \\ & & \cdots\cdots\cdots & & \\ 1 & x_{n-1} & {x_{n-1}}^2 & \cdots & {x_{n-1}}^{n-2} \end{vmatrix}$$

3. Let A be the matrix below. (You can quote the formula of the previous problem.)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2^2 & 2^3 & 2^4 \\ 1 & 3 & 3^2 & 3^3 & 3^4 \\ 1 & 4 & 4^2 & 4^3 & 4^4 \\ 1 & 5 & 5^2 & 5^3 & 5^4 \end{bmatrix}$$

(a) Show that A is invertible. (5 pts)

(b) Find the $(5, 1)$-entry of the inverse of A. (5 pts)

(c) Find the $(1, 4)$-entry of the inverse of A. (5 pts)

4. Let A, $\boldsymbol{x}$ and $\boldsymbol{b}$ be the matrices below.

$$A = \begin{bmatrix} 0 & 2 & -5 & 4 \\ -1 & -2 & 0 & 4 \\ 1 & -3 & -1 & 2 \\ 2 & -5 & -3 & 4 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

(a) Evaluate $\det(A)$. (10 pts)

(b) Applying the Cramer's rule to find x_4 of the equation $A\boldsymbol{x} = \boldsymbol{b}$. (10 pts)

5. Let B, C, $\boldsymbol{x}$ and $\boldsymbol{b}$ be matrices below.

$$B = \begin{bmatrix} 0 & 2 & -5 & 4 & a \\ -1 & -2 & 0 & 4 & b \\ 1 & -3 & -1 & 2 & c \\ 2 & -5 & -3 & 4 & d \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -3 & -1 & 2 & a' \\ 0 & 1 & -1 & 0 & b' \\ 0 & 2 & -5 & 4 & c' \\ 0 & -5 & -1 & 6 & d' \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

(a) By a consecutive application of elementary row operations, the matrix C is obtained from B. Express a', b', c' and d' in terms of a, b, c, d. (10 pts)

(b) Let P be a 4×4 matrix such that $PB = C$. Express P^{-1} as a product of elementary matrices using the notation $P(i;\alpha)$, $P(i,j)$, $P(i,j;\beta)$. (10 pts)

(c) Show that for any numbers b_1, b_2, b_3, b_4, the equation $B\boldsymbol{x} = \boldsymbol{b}$ has infinitely many solutions. (10 pts)

6. Let $A = \begin{bmatrix} x & 1 & 0 \\ 0 & x & 1 \\ 0 & 0 & x \end{bmatrix}$

(a) Find the matrices A^2 and A^3. (5 pts)

(b) Find the matrix A^n for any natural number $n = 1, 2, 3, \ldots$. (10 pts)

メッセージ欄：この授業について、その他何でもどうぞ。