Solutions to Take-Home Quiz 3 (*September 28, 2007*)

Let A and B be 3×3 matrices given below, and $C = [\,A \mid I\,]$, where I is the identity matrix of size three.

$$A = \begin{bmatrix} -3 & 1 & -1 \\ -3 & 1 & -2 \\ -1 & 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ -3 & 1 & -1 \end{bmatrix}, \; \text{and } C = \begin{bmatrix} -3 & 1 & -1 & 1 & 0 & 0 \\ -3 & 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & -2 & 0 & 0 & 1 \end{bmatrix}$$

We applied elementary row operations $[1,3]$, $[1;-1]$, $[2,1;3]$ to the matrix C in this order and obtained a matrix $[\,B \mid P\,]$, where B is a 3×3 matrix above and P is a 3×3 matrix.

1. Find the matrix P.

   Sol.

$$I \xrightarrow{[1,3]} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{[1;-1]} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{[2,1;3]} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & -3 \\ 1 & 0 & 0 \end{bmatrix} = P$$

2. Find the reduced row echelon form of the matrix C. (Solution only.)

   Sol.

$$[B \mid P] = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 & 1 & -3 \\ -3 & 1 & -1 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{[3,1;3]} \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 & 1 & -3 \\ 0 & 1 & 5 & 1 & 0 & -3 \end{bmatrix} \xrightarrow{[3,2;-1]}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 & 1 & -3 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{[1,3;-2]} \begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 4 & 0 & 1 & -3 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{[2,3;-4]}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & -4 & 5 & -3 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix} = [I \mid A^{-1}] \quad \text{(Reduced Echelon Form)}$$

3. Find the inverse matrix of A. (Solution only.)

   Sol.

$$A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ -4 & 5 & -3 \\ 1 & -1 & 0 \end{bmatrix}.$$

4. Express P^{-1} as a product of elementary matrices using the notation $P(i;c)$, $P(i,j)$ and $P(i,j;c)$.

   Sol. Since $P = P(2,1;3)P(1;-1)P(1,3)$,

$$P^{-1} = P(1,3)^{-1}P(1;-1)^{-1}P(2,1;3)^{-1} = P(1,3)P(1;-1)P(2,1;-3).$$