Solutions to Take-Home Quiz 1 (*September 10, 2010*)

$$\begin{cases} -2x_1+2x_2+5x_3+4x_4-9x_5-10x_6 &= -13 \\ -x_1+x_2+3x_3+3x_4-8x_5-7x_6 &= -6 \\ 2x_1-2x_2-4x_3-x_4-3x_5+4x_6 &= 14 \\ x_1-x_2-2x_3-x_4+x_5+3x_6 &= 7 \end{cases} \qquad B=\begin{bmatrix} 1 & -1 & -2 & -1 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 & -7 & -4 & 1 \\ 2 & -2 & -4 & -1 & -3 & 4 & 14 \\ -2 & 2 & 5 & 4 & -9 & -10 & -13 \end{bmatrix}$$

1. Find the augmented matrix A of the system of linear equations above.

   Sol.

$$A=\begin{bmatrix} -2 & 2 & 5 & 4 & -9 & -10 & -13 \\ -1 & 1 & 3 & 3 & -8 & -7 & -6 \\ 2 & -2 & -4 & -1 & -3 & 4 & 14 \\ 1 & -1 & -2 & -1 & 1 & 3 & 7 \end{bmatrix}$$

2. The matrix B is obtained by applying elementary row operations twice to the augmented matrix A. Write the elementary row operation using the notation $[i;c]$, $[i,j]$, or $[i,j;c]$.

   Sol. First apply $[1,4]$ and then $[2,1;1]$.

3. Find the reduced row echelon form of the augmented matrix A. (Solution only.)

   Sol. Apply the following consecutively in this order:

$$[3,1;-2],\ [4,1;2],\ [4,2;-1],\ [2,3;-2],\ [1,3;1],\ [1,2;2].$$

   Then we have

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 2 & 1 & 9 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & -5 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   - There are many ways to obtain the reduced echelon form but the final matrix should be the same. When can we change the order of operations and when cannot?
   - Starting from the reduced row echelon form above, is it possible to obtain the matrix A back again by applying elementary row operations? Can you find the sequence of such elementary row operations from the one we obtained the reduced echelon form from A with a slight modification?

4. Find the solution of the system of linear equations. Use parameters if necessary.

   Sol.

$$\begin{cases} x_1 &= 9+s-2t-u \\ x_2 &= s \\ x_3 &= 1-3t \\ x_4 &= 5t+2u \\ x_5 &= t \\ x_6 &= u \end{cases}, \text{ or } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ -3 \\ 5 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -1 \\ 0 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$

   s, t and u are parameters.