Division: **ID#:**   **Name:**

Let A, $\boldsymbol{x}$, $\boldsymbol{b}$, $\boldsymbol{c}$ be as follows.

$$A = \begin{bmatrix} -2 & 1 & 4 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & -3 & 1 & -1 \\ 1 & 3 & -2 & 1 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{c} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}.$$

1. Find a sequence of elementary row operations that transform $[A \mid I]$ to a reduced row echelon form. (Use $[i; c]$, $[i, j]$ and $[i, j; c]$ notation.) (Show work!)

2. Write A as a product of elementary matrices $P(i; c)$, $P(i, j)$, $P(i, j; c)$.

3. Show that for a given $\boldsymbol{b}$, $A\boldsymbol{x} = \boldsymbol{b}$ always has a unique solution.

4. Find the solution $\boldsymbol{x}$ of an equation $A\boldsymbol{x} = \boldsymbol{c}$.

Message 欄：将来の夢、目標、25年後の自分について、世界について。[HP 掲載不可は明記のこと]