# Solutions to Take-Home Quiz 5 (*October 15, 2010*)

Let $A$ and $B$ be the $5\times 5$ matrices given below.

$$A=\begin{bmatrix} 2 & 0 & -3 & 1 & 4\\ -2 & 2 & -1 & -3 & 2\\ -3 & -1 & 0 & 4 & 1\\ 2 & 2 & 1 & 3 & -2\\ -2 & -3 & 3 & -2 & 4\end{bmatrix},\quad B=\begin{bmatrix} a & b & b & b & b\\ b & a & b & b & b\\ b & b & a & b & b\\ b & b & b & a & b\\ b & b & b & b & a\end{bmatrix}.$$

1. Find the numbers $x$, $y$ and $z$. (Solutions only.)

$$\det(A)=x\cdot\begin{vmatrix} 2 & 2 & 1 & 3 & -2\\ 8 & 6 & 0 & 10 & -2\\ 0 & 4 & 0 & 0 & 0\\ -3 & -1 & 0 & 4 & 1\\ -8 & -9 & 0 & -11 & 10\end{vmatrix}=y\cdot\begin{vmatrix} 8 & 6 & 10 & -2\\ 0 & 1 & 0 & 0\\ -3 & -1 & 4 & 1\\ -8 & -9 & -11 & 10\end{vmatrix}=z\cdot\begin{vmatrix} 4 & 5 & -1\\ -3 & 4 & 1\\ -8 & -11 & 10\end{vmatrix}.$$

$$x=-1,\qquad y=-4,\qquad z=-8.$$

2. Find $\det(A)$. (Show work.)

**Sol.**

$$\det(A)=-8\cdot\begin{vmatrix} 4 & 5 & -1\\ -3 & 4 & 1\\ -8 & -11 & 10\end{vmatrix}\overset{[1,2;1],[3,2;-10]}{=}-8\cdot\begin{vmatrix} 1 & 9 & 0\\ -3 & 4 & 1\\ 22 & -51 & 0\end{vmatrix}=8\begin{vmatrix} 1 & 9\\ 22 & -51\end{vmatrix}$$

$=8(-51-9\cdot 22)=-8\cdot 249=-1992.$ ■

3. Explain why the following equalities hold.

$$\det(B)\overset{(1)}{=}\begin{vmatrix} a+4b & a+4b & a+4b & a+4b & a+4b\\ b & a & b & b & b\\ b & b & a & b & b\\ b & b & b & a & b\\ b & b & b & b & a\end{vmatrix}\overset{(2)}{=}(a+4b)\begin{vmatrix} 1 & 1 & 1 & 1 & 1\\ b & a & b & b & b\\ b & b & a & b & b\\ b & b & b & a & b\\ b & b & b & b & a\end{vmatrix}.$$

**Sol.** (1) Applied the following row operations. $[1,2;1]$, $[1,3;1]$, $[1,4;1]$, $[1,5;1]$ and the value of the determinant remains the same.

(2) Factor out $a+4b$ from the first row. ■

4. Find the condition that $B$ is invertible.

**Sol.** By applying the operations $[2,1;-b]$, $3,1;-b]$, $[4,1;-b]$, $[5,1;-b]$, we have

$$\det(B)=(a+4b)\begin{vmatrix} 1 & 1 & 1 & 1 & 1\\ b & a & b & b & b\\ b & b & a & b & b\\ b & b & b & a & b\\ b & b & b & b & a\end{vmatrix}=(a+4b)\begin{vmatrix} 1 & 1 & 1 & 1 & 1\\ 0 & a-b & 0 & 0 & 0\\ 0 & 0 & a-b & 0 & 0\\ 0 & 0 & 0 & a-b & 0\\ 0 & 0 & 0 & 0 & a-b\end{vmatrix}$$

$=(a+4b)(a-b)^4$. So $B$ is invertible if and only if $(a+4b)(a-b)\neq 0$ if and only if $a+4b\neq 0$ and $a-b\neq 0$. ■